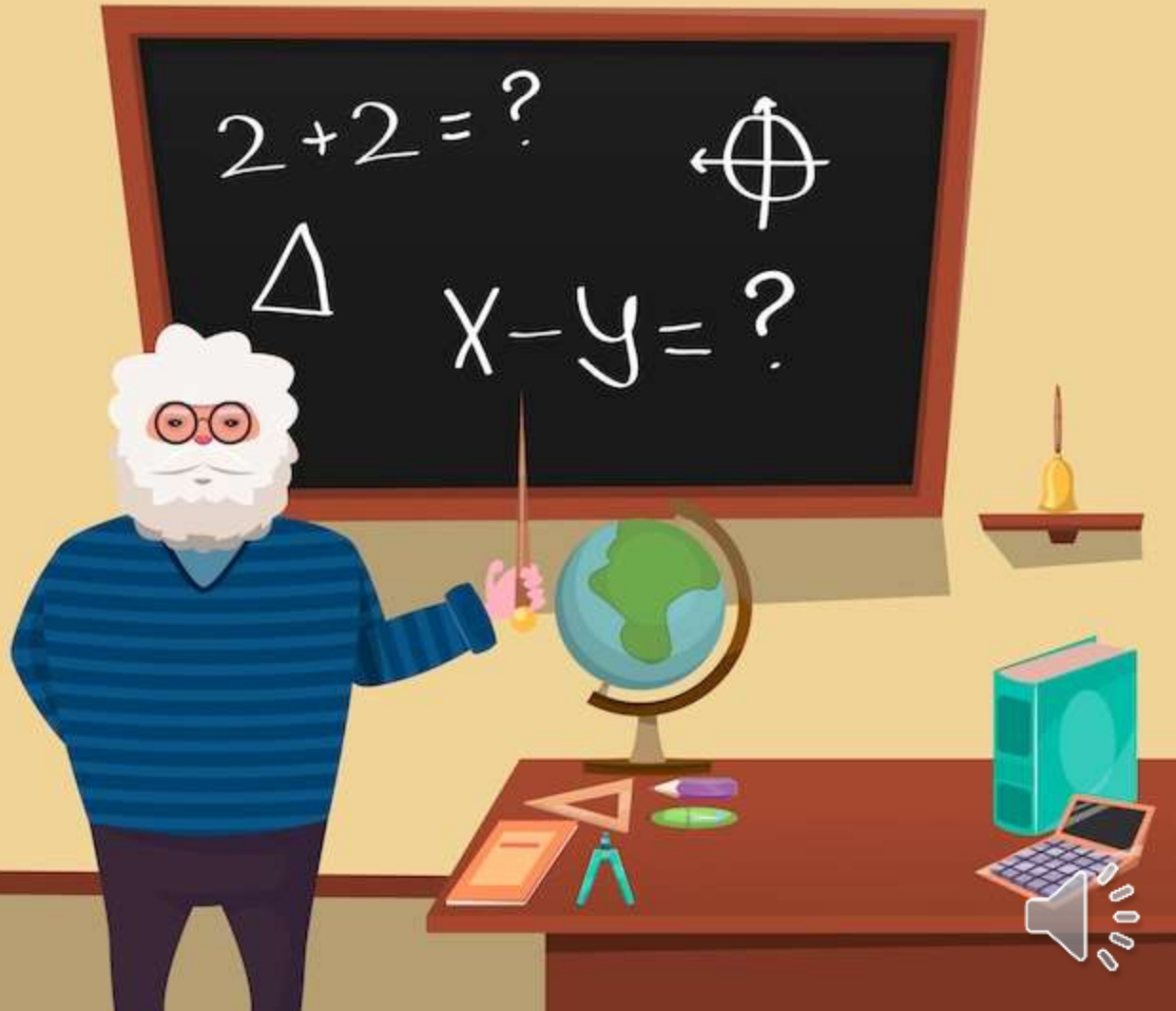
Proportion (Part 3)



► Introductory application

A market sells items. Sometimes the store make discount (sold) on price and sometimes make a raise. This mean that the price of the items change.

Example 1:

In black Friday, the store make a discount on prices. A shirt cost $25 before, after the discount the price becomes $20.

Notice that the price decreases $5.

This $5 is decreased from the original price $25.

$25 ⟶ 100%

$5 ⟶ x%

$$x = \frac{5 \times 100}{25} = 20\%$$

The price decreased 20%

► Introductory application

Example 2:

When summer comes, the store make a raise on prices of shirts. A shirt cost $25 before, after the raise the price becomes $30.

Notice that the price increases $5.

This $5 is increased on the original price $25.

$25 ⟶ 100%

$5 ⟶ x%

$$x = \frac{5 \times 100}{25} = 20\%$$

The price increased 20%

► Introductory application

In the two cases we have:

- ❖ Original price
- ❖ New price
- ❖ Percentage change

What are the relations between these 3?

► Raise in price

An item costs 25$. It raised 20%. What is its new price?

To find the new price, we need to find the amount of 20%.

20% of 25 is $\frac{20}{100} \times 25 = 5$

So the new price is 25 + 5 = 30.

Suppose that the original price is x , the new price is y and the percentage of change is a%.

- The amount of the percentage is $\frac{a}{100}$ of x i.e. $\frac{a}{100} \times x$
- The new price is $y = x + \frac{a}{100}x$

$$= x\left(1 + \frac{a}{100}\right)$$

$$= \left(1 + \frac{a}{100}\right)x$$

$$y = \left(1 + \frac{a}{100}\right)x$$

► Discount in price

An item costs 25$. It decreased 20%. What is its new price?

To find the new price, we need to find the amount of 20%.

20% of 25 is $\frac{20}{100} \times 25 = 5$

So the new price is 25 – 5 = 20.

Suppose that the original price is x , the new price is y and the percentage of change is a%.

- The amount of the percentage is $\frac{a}{100}$ of x i.e. $\frac{a}{100} \times x$
- The new price is $y = x - \frac{a}{100}x$

$$= x\left(1 - \frac{a}{100}\right)$$

$$= \left(1 - \frac{a}{100}\right)x$$

$$y = \left(1 - \frac{a}{100}\right)x$$

► Percentage of change

The percentage of change:

$$a = \frac{original\ price - new\ price}{original\ price} \times 100$$

If $a < 0$, there is raise in price.

If $a > 0$, there is discount on price.

Example:

- A pant costs 30$. After reducing its price, it becomes 27$

$a = \frac{30-27}{30} \times 100 = 10\%$ so the percentage of change is 10%

- A pant costs 30$. After raising its price it becomes 36$.

$a = \frac{30-36}{30} \times 100 = -20\%$ so the percentage of change is 20%.

► Application 1

The price of an article is \$150. its price increases by 9% what is the new price?

$$y = \left(1 + \frac{a}{100}\right)x$$
$$= \left(1 + \frac{9}{100}\right) \times 150$$
$$= 1.09 \times 150 = 163.5$$

The new price is \$163.5

► Application 2

The price of an object after a discount of 10% is \$550. what is its original price?

$$y = \left(1 - \frac{a}{100}\right)x$$

$$550 = \left(1 - \frac{12}{100}\right)x$$

$$550 = 0.88x$$

$$x = \frac{550}{0.88} = 625$$

The original price is \$625.

► Application 3

The price of an object decreases by 15% and then increases by 20%. If its original price was 75$, what is its new price?

After decrease of 15%:

Original price is 75$

$$y = \left(1 - \frac{15}{100}\right) x = 0.85 \times 75 = 63.75\$$$

After an increase of 20%:

Original price is 63.75$

$$z = \left(1 + \frac{20}{100}\right) y = 1.2 \times 63.75 = 76.5\$$$

Or

x is the original price and y its new price after the two changes;

$$y = \left(1 - \frac{15}{100}\right)\left(1 + \frac{20}{100}\right) x$$

$$= 0.85 \times 1.2 \times 75 = 76.5\$$$

► Application 4

The price of an article was 75$, it becomes 125$. What is the percentage of change?

Original price: 75$

New price: 125$

Percentage of change $a = \frac{original\ price - new\ price}{original\ price} \times 100$

$$= \frac{75-125}{75} \times 100 = -66.67\%$$

So there is increase in price by approximately 66.67%

► Application 5

The price of an article increased by 15% then decreased by 15%. What is the change that is happened on the price?

Original price: x

New price: y

$$y = \left(1 + \frac{15}{100}\right)\left(1 - \frac{15}{100}\right)x$$

$$= 0.9775x$$

$0.9775 < 1$ so there is decrease in price

$$1 - \frac{a}{100} = 0.9775$$

$$1 - 0.9775 = \frac{a}{100}$$

$$0.0225 = \frac{a}{100} \text{ so } a = 2.25\%$$